International Journal of Research in Advent Technology, Vol.6, No.3, March 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

On Non-Homogeneous Sextic Equation with Five Unknowns

R.Anbuselvi[1],S.A. Shanmugavadivu[2]
[1]*Professor, Department of Mathematics, A.D.M. College for Women(Autonomous), Nagapattinam-611001, Tamilnadu, India.*
[2]*Professor, Department of Mathematics, T.V.K. Govt Arts College, Tiruvarur -610003, Tamilnadu, India.*

Abstrac**t-** The non-homogeneous sextic equation with five unknowns given by $(2x+2y)(2x^3-2y^3)=32(2z^2-2w^2)T^4$ is considered and analysed for its non-zero distinct integer solutions. Employing the linear transformations$x=2u+2v, y=2u-2v, z=4u+v,\ w=4u-v, (u\neq v\neq 0)$ and applying the method of factorization, three different patterns of non-zero distinct integer solutions are obtained. A few interesting relation between the solutions and special numbers.

Keywords: Integer solutions, Non-homogeneous sextic equations with five unknowns.

1. INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems [1-4] particularly, in [5, 6] sexticequation with three unknowns are studied for their integral solutions [7, 12] analyze Sextic equations with five unknowns for their non-zero integer solution. This communication analyzes a Sextic equations with five unknowns given by $(2x+2y)(2x^3-2y^3)=32(2z^2-2w^2)T^4$. Infinitely many Quintuples (x,y,z,w,T). Satisfying the above equation is obtained. Various interesting properties among the values of $x,y,z,w\ and\ T$ are presented.

2. NOTATIONS

- ❖ $t_{m,n}$ =Polygonal number of rank n with size m
- ❖ P_n^m = pyramidal number of rank n with size m
- ❖ P_r^n = pronic number of rank n
- ❖ SO_n= Stella octangular number of rank n
- ❖ j_n= jacobthallucas number of rank n
- ❖ J_n= Jacobthal number of rank n
- ❖ Gno_n= Gnomic number of rank n
- ❖ Cp_n^6 = Centered pyramidal number of rank n with size m
- ❖ Cp_n^{14} = Centered tetra decagonal pyramidal number of rank n
- ❖ Ky_n =Kynea number of rank n.
- ❖ Obl_n = Oblong Number of rank n
- ❖ CH_n = Centered Hexagonal number of rank n.
- ❖ CP_n = Centered pentagonal number of rank n.
- ❖ S_n = Star number of rank n.
- ❖ $4DF_n$= Four Dimensional figurate number whose generating polygon is a square.

3. METHOD OF ANALYSIS

The non-homogeneous sextic equation with five unknowns to be solved is given by.
$(2x+2y)(2x^3-2y^3)=32(2z^2-2w^2)T^4$ -------------------(1)

The substitution of the linear transformations
$x=2u+2v, y=2u-2v, z=4u+v, w=4u-v, (u\neq v\neq 0)$----------------(2)
In (1) leads to
$v^2+3u^2=4T^4$----------------(3)
(3) is solved through different approaches and different patterns of solutions thus obtained for (1) are illustrated below:

Pattern:1

Assume $T=T(a,b)=a^2+3b^2,\ a,b>0$--------------(4)
Write 4 as
$4=(1+i\sqrt{3})(1-i\sqrt{3})$--------------(5)
Using (4) and (5) in (3) and employing the method of factorization and equating positive factors, we get
$v+i\sqrt{3}u=(1+i\sqrt{3})(a+i\sqrt{3}b)^4$
Equating real and imaginary parts, we have
$u=u(a,b)=a^4+9b^4-18a^2b^2+4a^3b-12ab^3$

$v = v(a,b) = a^4 + 9b^4 - 18a^2b^2 - 12a^3b + 36ab^3$

Employing (2), the values of x,y,z, w ant T are given by

$x = x(a,b) = 4a^4 + 36b^4 - 72a^2b^2 - 16a^3b + 52ab^3$
$y = y(a,b) = 32a^3b - 96ab^3$
$z = z(a,b) = 5a^4 + 45b^4 - 90a^2b^2 + 4a^3b - 12ab^3$
$w = w(a,b) = 3a^4 + 27b^4 - 54a^2b^2 + 28a^3b - 84ab^3$
$T = T(a,b) = a^2 + 3b^2$

Which represent non zero distinct integer solutions of (1) in two parameters.

Properties

- $T(1,2^n) + 9J_n + 3j_n - 4 = 3ky_n$
- $x(a,1) - 4(T_{4,a})^2 - 16CP_a^6 + 72T_{4,a} \equiv 36(mod\ 52)$
- $w(a,1) + T(a^2,1) - 4Fn_a^4 + 50T_{4,a} + 2816CP_a^6 - 30 = 0$
- $T(a^2,a^2) - 4(T_{4,a})^2 = 0$
- $y(1,b) - 3CP_b^4 - 67CP_b^6 + 72CP_b^3 \equiv 0(mod\ 72)$
- $y(a,1) - z(a,1) - w(a,1) + 384DF_a - 136\ Obl_a \equiv 72(mod\ 136)$
- $T(a,a+1) - T_{4,a} - Pr_a - 3Gno_a = 6$
- $z(1,b) - 180DF_b + 24P_b^5 + CH_b - 36\ Obl_b \equiv 4(mod\ 43)$
- $7z(1,b) - w(1,b) - 3456FN_b^4 + 2(T_{4,12b}) - j_4 - 3J_4 = 0$
- $T(b(b+1),b+1) - 12FN_b^4 - 2CP_b - SO_b \equiv 1(mod\ 2)$

Pattern:2

One may write (3) as

$v^2 + 3u^2 = 4T^4 * 1$------------------(6)

Also, write 1 as

$1 = \frac{(2+i2\sqrt{3})(2-i2\sqrt{3})}{16}$------------------(7)

Substituting (4), (5) and (7) in (6) and employing the method of factorization and equating positive factors we get

$v + i\sqrt{3}u = (1+i\sqrt{3})\frac{(2+i2\sqrt{3})}{4}(a + i\sqrt{3}b)^4$

Equating real and imaginary parts, we have

$u = u(a,b) = a^4 + 9b^4 - 18a^2b^2 - 4a^3b + 12ab^3$
$v = v(a,b) = -a^4 - 9b^4 + 18a^2b^2 - 12a^3b + 36ab^3$

In view of (2), the integer values of x,y,z,w and T are given by

$x = x(a,b) = -32a^3b + 96ab^3$
$y = y(a,b) = 4a^4 + 36b^4 - 72a^2b^2 + 16a^3b - 48ab^3$
$z = z(a,b) = 3a^4 + 27b^4 - 54a^2b^2 - 28a^3b + 84ab^3$
$w = w(a,b) = 5a^4 + 45b^4 - 90a^2b^2 - 4a^3b + 12ab^3$
$T = T(a,b) = a^2 + 3b^2$

Which represent non-zero distinct integer solutions of (1) in two parameters

Properties

- $3y(a,1) - 4z(a,1) - 480SO_a - 800CP_a^6$
- $3x(a,a) + 16T(a^2,\ a^2)$ is a biquadratic number
- $w(a,1) + 90T(a,1) - 5(T_{4,a})^2 + 8CP_a^3 + 16Gno_a - j_8 = 74$
- $z(a,1) - 3T(a^2,1) + 12CP_a^{14} + 54(T_{4,a}) \equiv 18\ (mod\ 68)$
- $x(1,\ b) - 36Biq_b - 52Cub_b + 18T_{4,2b} \equiv 4(mod\ 16)$
- $x(1,b) - z(1,b) - w(1,b) + 864FN_b^4 - 2(T_{4,6b}) + j_3 = 1$
- $T(2a+1,2a) - T_{13,a} + 5\ Obl_a + 4Gno_a + j_2$
- $x(b,b+1) - 384FN_b^4 - 32{T_{4,b}}^2 - 64\ Obl_b - 512P_b^5 \equiv 0(mod\ 32)$
- $z(a+1,a) - 29Biq_a - 105CP_a^6 + 138T_{4,a} - 9\ Obl_a + j_3 + j_5 = 0$
- $w(1,b) - 5T(1,b) - 45{T_{4,b}}^2 - 18\ OH_b + 95T_{4,b} + 10\ Obl_b$

Pattern:3

Write (3) as

$$3(u^2 - T^4) = T^4 - v^2 \quad (10)$$

Factorizing (10) we have

$$3\{(u+T^2)(u-T^2)\} = (T^2+v)(T^2-v) \quad (11)$$

This equation is written in the form of ratio as

$$\frac{3(u-T^2)}{T^2-v} = \frac{T^2+v}{u+T^2} = \frac{a}{b}, \quad b \neq 0 \quad (12)$$

Which is equivalent to the system of double equations

$$3bu + av - (3b+a)T^2 = 0 \quad (13)$$

$$-au + bv + (b-a)T^2 = 0 \quad (14)$$

Applying the method of cross multiplication, we get

$$u = -a^2 + 3b^2 + 2ab \quad (15)$$

$$v = a^2 - 3b^2 + 6ab \quad (16)$$

$$T^2 = a^2 + 3b^2 \quad (17)$$

Now, the solution for (17) is

$$a = 3p^2 - q^2, b = 2pq, \ T = 3p^2 + q^2 \quad (18)$$

Using (18) in (15) and (16), we get

$$u = u(p,q) = -9b^4 - q^4 + 18p^2q^2 + 12p^3q - 4pq^3$$

$$v = v(p,q) = 9p^4 + q^4 - 18p^2q^2 + 36p^3q - 12pq^3$$

In view of (2), the integer values of x,y,z,w and T are given by

$$x = x(p,q) = 2u + 2v = 96p^3q - 36pq^3$$

$$y = y(p,q) = 2u - 2v = -36p^4 - 4q^4 + 72p^2q^2 - 48p^3q + 16pq^3$$

$$z = z(p,q) = 4u + v = -27p^4 - 3q^4 + 54p^2q^2 + 84p^3q - 28pq^3$$

$$w = w(p,q) = 4u - v = -48p^4 - 5q^4 + 90p^2q^2 + 12p^3q - 4pq^3$$

$$T = T(p,q) = 3p^2 + q^2$$

Which represent non-zero distinct integer solutions of (1) in two parameters

Properties

- $x(p,1) + 2y(p,1) + 864FN_p^4 - 72T_{4,p} + j_3 = 1$
- $3x(p,p) + 16T(p^2, \ p^2)$ is a biquadratic number
- $w(p,1) - 30T(p,1) + 45(T_{4,p})^2 - 4SO_p + 4CP_n^6 + 35 = 0$
- $T(1,2)$ is mersenne primes and perfect number
- $x(p,1) - 48SO_p \equiv 0 (mod\ 16)$
- $y(1,q) + 48FN_q^4 - 24OH_q - T_{24,q} - 57T_{4,q} \equiv 36 (mod\ 46)$
- $T(2p, 2p+2) - S_p - 10Pr_b - 2Gno_p - j_2$
- $x(1,q) + T(1,q) + 16SO_q - Obl_q - Gno_q \equiv 4 (mod\ 77)$
- $z(1,q) + 3T_{4,q2} - 28CP_q^6 - 54Pr_q - 15Gno_q + j_3 = 5$

4. CONCLUSION

First of all, it is worth to mention here that in (2), the values of z and w may also be represented by $z = uv + 4$, $w = uv - 4$ and $z = 2uv + 2$, $w = 2uv - 2$ and thus, will obtain other choices of solutions to (1). In conclusion, one may consider other forms of Sextic equation with five unknowns and search for their integer solutions.

REFERENCES

[1] Dickson, L.E., History of theory of numbers, vol.2, *Chelsea publishing company*, *Newyork*(1952).

[2] Carmichael, R.D., The theory of numbers and Diophantine analysis, *Dover publications*, *Newyork*(1959).

[3] Mordell, L.J., Diophantine equations, *Academic press*, *London* (1969).

[4] Telang, S.G., Number Theory, *Tata MC Graw Hill Publishing Company*, *New Delhi* (1996).

[5] Gopalan, M.A., ManjuSomanath and Vanitha, N., "Parametric Solutions of $x^2 - y^6 = z^2$ ", *ActaCienciaIndica XXXIII*, vol.3, 1083-1085, 2007.

[6] Gopalan, M.A., Sangeetha, G., "On the Sextic Equations with three unknowns $x^2 - xy + y^2 = (k^2 + 3)^n z^6$",Impact *J.Sci.tech*, Vol.4, No: 4, 89-93, 2010.
[7] Gopalan, M.A., Vijayashankar, A., "Integral solutions of the SexticEquation$x^4 + y^4 + z^4 = 2w^6$ ", *Indian journal of Mathematics and Mathematical Sciences*, Vol.6, No:2, 241-245, 2010.
[8] Gopalan, M.A., Vidhyalakshmi, S., Vijayashankar, A., "Integral Solutions of Non-Homogeneous Sexticequation$xy + z^2 = w^6$", *Impact J.Sci.tech*, Vol.6, No: 1, 47-52, 2012.
[9] Gopalan, M.A., Vidhyalakshmi, S., Lakshmi, L., "On the Non-Homogeneous SexticEquation $x^4 + 2(x^2 + w)x^2y^2 + y^4 = z^4$" , *IJAMA*, 4(2), 171-173, Dec (2012).
[10] Gopalan, M.A., Vidhyalakshmi, S., Kavitha, A., "Observations on the Homogeneous Sextic Equation with four unknowns$x^3 + y^3 = 2(k^2 + 3)z^5 w$ ", *International Journal of Innovative Research in Science, Engineering and Technology*, Vol.2, Issue: 5, 1301-1307, 2013.
[11] Gopalan, M.A., Sumathi, G., Vidhyalakshmi, S., "Integral Solutions of Non- homogeneous Sextic Equation with four unknowns $x^4 + y^4 + 16z^4 = 32w^6$", *Antarctica J.Math*, 10(6), 623-629, 2013.
[12] Gopalan, M.A., Sumathi, G., Vidhyalakshmi, S., "Gaussian Integer Solutions of Sextic Equations with four unknowns $x^6 - y^6 = 4z(x^4 + y^4 + w^4)$", *Archimedes, J.Math*, 3(3), 263-266, 2013.
[13] Gopalan, M.A., Vidyalakshmi, S., Lakshmi, K., "Integral Solutions of Sextic Equation with Five unknowns $x^3 + y^3 = z^3 + w^3 + 3(x - y)t^5$" , *IJERST*, 1(10), 562-564, 2012.
[14] Gopalan, M.A., Sumathi, G., Vidyalakshmi, S., "Integral Solutions of Sextic Non-Homogeneous Equation with Five unknowns $x^3 + y^3 = z^3 + w^3 + 6(x + y)t^5$", *International Journal of Engineering Research*, Vol.1, Issue.2, 146-150, 2013.
[15] Gopalan, M.A., AarthyThangam, S., Kavitha, A., "On Non-homogeneous Sextic equation with five unknowns$2(x - y)(x^3 + y^3) = 28(z^2 - w^2)T^4$", *Jamal Academic Research Journal (JARJ)* , special Issue, 291-295, ICOMAC- 2015.
[16] Meena, K., Vidhyalakshmi, S., AarthyThangam, S., "On Non-homogeneous Sextic equation with five unknowns $(x + y)(x^3 + y^3) = 26(z^2 - w^2)T^4$", *Bulletin of Mathematics and Statistics Research*, Vol.5, Issue.2, 45-50, 2017.